1 Conceptual Review

Strategic game (w/ complete info)

Def: $\langle N, (A_i), (u_i) \rangle$

Strategy: $s_i \in A_i$

$\sigma_i \in \Delta(A_i)$

Solution concept:

ESS ← only for symmetric games

$\subseteq$

NE

$\subseteq$

Correlated Eq

σ is THPE if $\exists\, \sigma^k \to \sigma$ (non-constant), $\sigma_i \in BR_i(\sigma_{-i}^k)$ $\forall i, k$

Trembling Hand Perfect Eq

$\subseteq$

NE

σ must be NE (in 2 player game, THPE = mixed NE w/ no probability on weakly dominated actions)

$$\langle N, (A_i), (u_i), \underbrace{\Omega, \pi}_{\text{probability space}}, \underbrace{\tilde{P}_i}_{\text{partition over states}} \rangle$$

Strategy $\sigma_i : \tilde{P}_i \to A_i$

$\Updownarrow$

($\sigma_i : \Omega \to A_i$, $\sigma_i(w) = \sigma_i(w')$ if $w, w' \in P_i$)

Thm: WLOG, $\Omega = \prod A_i$, $\tilde{P}_i$ consists of action profiles where i takes the same action.

Bayesian game

(Static game w/ incomplete info)

$$\langle N, \Omega, (A_i), \underbrace{(T_i)}_{\text{type space}}, \underbrace{(\tau_i)}_{\text{signaling fcn}}, \underbrace{(p_i)}_{\text{prior over } \Omega}, (u_i) \rangle$$

Strategy: $\sigma_i : T_i \to \Delta(A_i)$

Solution concept: BNE

Two interpretations:

① (i, t_i) as set of players

$\underbrace{a^*}_{\text{of length } \sum_i |T_i|} \succsim b^* \iff L_i(a^*, t_i) \succsim L_i(b^*, t_i)$

② $\sigma_i : T_i \to \Delta(A_i)$

$$U_i(a_i, \sigma_{-i} \mid t_i) = \sum_{w, t_{-i}} u_i(w, a_i, \sigma_{-i}(t_{-i}))\, p(w, t_{-i} \mid t_i)$$

$t_i \to p(w, t_{-i}) \to p(w, \sigma_{-i}(t_{-i}))$

Repeated game

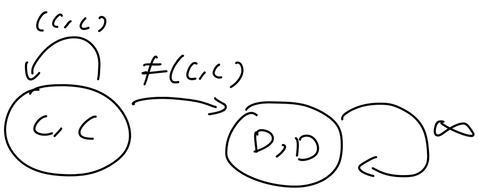
perfect info

SPE

grim-trigger

limited punishment

tit-for-tat



check one-stage deviation

Imperfect public monitoring

PPE

only using public strategies

Def: enforceable

generated

self-generating

We can find the set of PPE payoffs w/ self-generating sets

Imperfect private monitoring

private signal
- independent
- correlated

Folk Thms

: Set of enforceable payoffs when $\delta \to 1$.

Minimax payoff:

$$\underline{v_i} = \min_{s_{-i}} \left[\max_{s_i} g_i(s_i, s_{-i}) \right]$$

Nash Folk Thm — NE

"Nash threat" Folk Thm

Fudenberg & Maskin Folk Thm

} SPE

Extensive game (perfect info)

$< N, H, P, f_c, (u_i) >$

$A_i(h) = \{ a : (h, a) \in H \}$

Strategy (pure) $S_i : H \setminus Z \to A(h)$

(terminal history)

Solution concept: SPE ← If finite Γ, all SPE can be found w/ Backward induction

∴ NE for any subgame $\Gamma(h), \forall h$.

One-stage deviation principle

Notable example: Rubenstein Bargaining Game

SE = PBE + Consistency

$\exists (\beta^n, \mu^n) \to (\beta, \mu)$

β^n completed mixed

$\mu^n \xleftarrow[\text{rule}]{\text{Baye's}} \beta^n$

Trembling-Hand Perfect Eq in Agent-strategic form $\subseteq$ SE $\subseteq$ PBE

Extensive game (Imperfect info)

$< N, H, P, f_c, (\mathcal{I}_i), (u_i) >$

↑ information partition of H

$I_i \in \tilde{\mathcal{I}}_i$ information set

Strategy (pure) $S_i : \mathcal{I}_i \to A(I_i)$

- Mixed $\sigma_i \in \Delta(S_i(I_i))_{I_i}$
- Behavioral $(\beta_i : I_i \to \Delta(A_i))_{I_i}$

e.g. $\sigma_i = \frac{1}{2} HH + \frac{1}{2} HL$

$\beta_i = (\frac{1}{2}H + \frac{1}{2}L, \frac{1}{2}H + \frac{1}{2}L)$

$\cup$

Bayesian extensive game

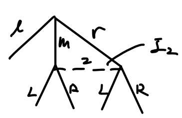
$< N, H, P, (\Theta_i), (p_i), (u_i) >$

Type space; prior over types (for now, assumed to be independence across i)

Strategy (pure) $S_i : \Theta_i \times H \to A_i(h)$

(behavioral) $(\beta_i : \Theta_i \times H \to \Delta A_i(h))_{I_i}$

Necessity of beliefs for an equilibrium



$\mu(m) + \mu(r) = 1$

belief within an information set

Assessment: $((\beta_i), (\mu_i))$

↑ Behavioral strategies ↑ Beliefs

PBE

1. Sequentially rational
   (β_i is best response given μ_i, β_{-i} for every information set I_i)
2. Bayesian updating whenever possible (when I_i is reached)
   PBE puts no restriction on off-path beliefs
3. Action determine beliefs
   beliefs on i's type can only be changed by i's action
   True only when types are independence.