ECON 6110, Microeconomic Theory III	**Section 7**
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1 Bargaining and the uniqueness

Question 2: Show that the equilibrium above is the unique SPE.

Proof. Instead of strategies, we consider the possible SPEs in terms of the equilibrium payoffs.

Let m_i and M_i be the infimum and supremum payoffs obtained by i in any SPE **as a proposer**. We can argue that:

$$m_i \geq 1 - \delta_j M_j \tag{1}$$

for $i = A, B$. Since $\delta_j M_j$ is the highest amount i should offer j (and to which j must accept). Similarly, we can argue that:

$$M_j \leq \max \left\{ \begin{array}{c} 1 - \delta_i m_i, \\ \delta_j(\delta_j M_j) \end{array} \right\}$$

Here, $\delta_i m_i$ is the lowest offer i could accept today, so $1 - \delta_i m_i$ is the highest possible payoff when j is the proposer **and** i accepts his proposal. On the other hand, if j makes an unacceptable offer, the max amount she can be offered tomorrow is $\delta_j M_j$. So j's discounted payoff today is no more than $\delta_j(\delta_j M_j)$.

Note that it must be:

$$\max \left\{ \begin{array}{c} 1 - \delta_i m_i, \\ \delta_j(\delta_j M_j) \end{array} \right\} = 1 - \delta_i m_i$$

Otherwise, we would have

$$M_j \leq \delta_j^2 M_j$$

which is only true if $M_j \leq 0$. However, if that's the case, we must have $1 - \delta m_i > \delta_j^2 M_j$, since $\delta, m_i < 1$ and $\delta_j^2 M_j < 0$, a contradiction. We conclude that:

$$M_j \leq 1 - \delta_i m_i \tag{2}$$

Lets put together (1) and (2) to obtain:

$$\begin{aligned} M_j &\leq 1-\delta_i m_i \\ &\leq 1-\delta_i(1-\delta_j M_j) \\ &\leq 1-\delta_i+\delta_i\delta_j M_j \\ &\Leftrightarrow M_j \leq \frac{1-\delta_i}{1-\delta_i\delta_j} \end{aligned}$$

Similarly, we can show that

$$\begin{aligned} m_j &\geq 1-\delta_i M_i \\ &\geq 1-\delta_i(1-\delta_j m_j) \\ &\geq 1-\delta_i+\delta_i\delta_j m_j \\ &\Leftrightarrow m_j \geq \frac{1-\delta_i}{1-\delta_i\delta_j} \end{aligned}$$

So

$$v_j = m_j = M_j = \frac{1-\delta_i}{1-\delta_i\delta_j}$$

This shows that the equilibrium payoffs are uniquely defined. This implies that the strategies must also be uniquely defined as

$$\begin{aligned} \alpha_i = v_i &= \frac{1-\delta_j}{1-\delta_i\delta_j} \\ 1-\alpha_i = \delta_j v_j &= \frac{\delta_j(1-\delta_i)}{1-\delta_i\delta_j} \end{aligned}$$

□