ECON 6110, Microeconomic Theory III	Section 13
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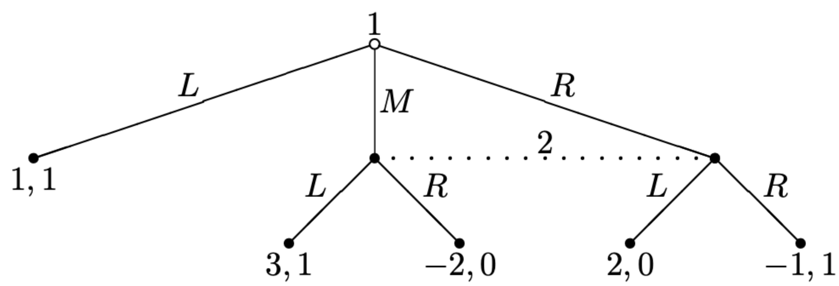
1 Sequential Equilibrium (SE)

Definition 1.1. An assessment (β, μ) is **consistent** if there is a sequence $((\beta^n, \mu^n))_{n=1}^{\infty}$ of assessments that converges to (β, μ) in Euclidian space and that

1. Each strategy profile β^n is completely mixed,
2. Each belief system μ^n is derived from β^n using Bayes' rule.

Definition 1.2. An assessment (β, μ) is a **sequential equilibrium** of a finite extensive game with perfect recall if it is sequentially rational and consistent.

Example 1.1.



Find the set of sequential equilibria of the game.

Solution: Since SE is a refinement of PBE, we can find all PBEs first, and then check if any of them still holds to be a SE.

You should find a PBE as follows:

$$\beta = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad \text{and} \quad \mu = \left(\begin{array}{cc} M & R \\ 1 & 0 \end{array}\right)$$

Consider the sequence of totally mixed strategies:

$$\beta^n = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 1/n & 1-2/n & 1/n & 1-1/n & 1/n \end{array}\right) \quad \text{and} \quad \mu^n = \left(\begin{array}{cc} M & R \\ \frac{1-2/n}{1-1/n} & \frac{1/n}{1-1/n} \end{array}\right)$$

We can verify that since $\frac{1-2/n}{1-1/n} = 2 - \frac{1}{1-1/n} \to 1$, (β, μ) is consistent so it is also SE.

You should find another class of PBEs $(1 > \delta > 0)$:

$$\beta = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 1 & 0 & 0 & \delta & 1-\delta \end{array} \right) \quad \text{and} \quad \mu = \left(\begin{array}{cc} M & R \\ 1/2 & 1/2 \end{array} \right)$$

Consider the sequence of totally mixed strategies:

$$\beta^n = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 1-2/n & 1/n & 1/n & \delta & 1-\delta \end{array} \right) \quad \text{and} \quad \mu^n = \left(\begin{array}{cc} M & R \\ 1/2 & 1/2 \end{array} \right)$$

We can verify that since $\beta^n \to \beta$ and $\mu^n = \mu$, so (β, μ) is consistent so it is also SE.

You should find yet another class of PBEs $(p \leq 1/2)$:

$$\beta = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 1 & 0 & 0 & 0 & 1 \end{array} \right) \quad \text{and} \quad \mu = \left(\begin{array}{cc} M & R \\ p & 1-p \end{array} \right)$$

Consider the sequence of totally mixed strategies:

$$\beta^n = \left(\begin{array}{ccc|cc} L & M & R & L & R \\ 1-1/n & p/n & (1-p)/n & 1/n & 1-1/n \end{array} \right) \quad \text{and} \quad \mu^n = \left(\begin{array}{cc} M & R \\ p & 1-p \end{array} \right)$$

We can verify that since $\beta^n \to \beta$ and $\mu^n = \mu$, so (β, μ) is consistent so it is also SE.

2 Trembling Hand Perfect Equilibrium

Sequential equilibrium restricts the player to hold "reasonable" off-path beliefs.

Trembling Hand Perfect Equilibria take a different route: each player allows the other players to make uncorrelated mistakes (their hands may tremble) that lead to off-path events. Moreover, each player's equilibrium strategy must be **robust** to such small mistakes of the other players. The essence of this concept is captured in static games.

Definition 2.1. A **trembling hand perfect equilibrium** of a finite strategic game is a mixed strategy profile σ with the property that there exists a sequence $(\sigma^k)_{k=0}^{\infty}$ of completely mixed strategy profiles that converges to σ such that for each player i the strategy σ_i is a best response to σ_{-i}^k for all values of k.

Remark. Trembling hand perfect equilibrium requires that σ_i is a best response to σ_{-i}^k, but this implies σ_i is a best response to σ_{-i}. This is because expected utility is linear in σ_{-i}^k, so

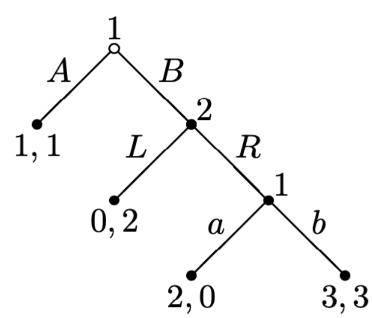
$$\mathbb{E}[u_i(\sigma_i, \sigma_{-i}^k)] - \mathbb{E}[u_i(\sigma_i', \sigma_{-i}^k)] \geq 0 \quad \forall \sigma_i', k$$

implies

$$\mathbb{E}[u_i(\sigma_i, \sigma_{-i})] - \mathbb{E}[u_i(\sigma_i', \sigma_{-i})] \geq 0$$

Thus, trembling hand perfect equilibrium $\subseteq$ Nash equilibrium.

Remark. When applying the solution concept in extensive games, we need to also allow the possibility a player's past and future selves to make mistakes. We study the trembling hand perfect equilibria of the **agent strategic form of the game**, in which there is one agent for each information set belonging to the same player.



In the agent strategic form of this game, $((A, a), L)$ is not a trembling hand perfect equilibrium since for any pair of completely mixed strategies of player 1's first agent and player 2, the unique best response of player 1's second agent is the pure strategy b.

Proposition 2.1. *For every trembling hand perfect equilibrium β of a finite extensive game with perfect recall there is a belief system μ such that (β,μ) is a sequential equilibrium of the game.*

Remark (Comparison with sequential equilibrium).