ECON 6110, Microeconomic Theory III	Section 11
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1 Bayesian Extensive Games and the Perfect Bayesian Equilibrium (PBE)

Definition 1.1. A **Bayesian extensive game with observed actions** is a tuple $\langle N, H, P, (\Theta_i), (p_i), (u_i)\rangle$ where:

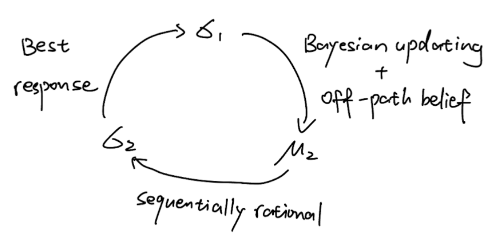
1. Set of N players, set of histories H, and player function P.
2. For each i:
   (a) A finite set of types Θ_i.
   (b) A probability measure p_i over Θ_i. (Assume independent types and common prior)
   (c) A preference relation $\succsim_i$ over $Z \times \Theta$.

Remark. In solving the game, we often recast the game as an extensive game with imperfect information, which is a tuple $\langle N, H, P, f_c, (\mathcal{I}_i), (u_i)\rangle$. We introduce Nature as another player, selecting types at time 0. (It will become clearer in the signaling game)

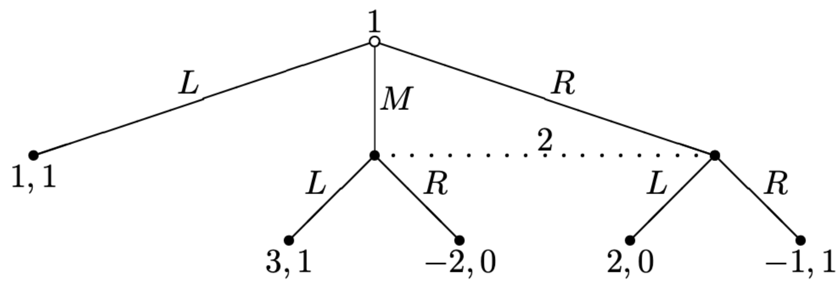
Definition 1.2 (Informal)**.** An assessment (σ, μ) is a **perfect Bayesian equilibrium** if

1. Sequentially rational: For each type θ_i, σ_i is the best response given μ_i and σ_{-i} at every information set I_i.
2. Bayesian updating whenever possible. (But no restriction on off-path beliefs)
3. Action determine beliefs: beliefs on i's type can only be changed by i's action. (True when independent types)

Remark. Solving for PBE often proceeds in a "loop":



2 Exercises



Find the set of perfect Bayesian Equilibria of the game.