ECON 6110, Microeconomic Theory III	Section 8
TA: Ruqing Xu	March 14, 2025

1 Folk Theorems

See Fudenberg and Tirole Chapter 5.1 for reference.

The "folk theorems" for repeated games assert that if the players are sufficiently patient, then any feasible, individually rational payoffs can be enforced by the equilibrium. To make this assertion precise, we must define "feasible" and "individually rational."

Definition 1.1. The set of **feasible payoffs** is the convex hull of payoffs generated by the pure action profiles in the stage game.

Definition 1.2. Player i's minimax payoff is:

$$\underline{v_i} = \min_{s_{-i}}[\max_{s_i} g_i(s_i, s_{-i})]$$

where s_i and s_{-i} are mixed strategies.

Remark. This is the lowest payoff player i's opponent can hold him to by any choice of s_{-i}, provided that player i correctly foresees s_{-i} and plays a best response to it.

Example 1.1. Find the minimax payoff for the following game.

	L	R
U	$-2, 2$	$1, -2$
M	$1, -2$	$-2, 2$
D	$0, 1$	$0, 1$

Answers: Player 1's minimax payoff: we first compute his payoffs to U, M, D as a function of the probability q that player 2 assigns to L.

$$v_U(q) = -3q + 1$$
$$v_M(q) = 3q - 2$$
$$v_D(q) = 0$$

Plot the max of these functions and find the minimum. We have that $\underline{v_1} = 0$.

Player 2's minimax payoff: we first compute his payoffs to L, R as a function of the probability x, y that player 1 assigns to U, M.

$$v_L(x, y) = 2(x - y) + (1 - x - y)$$
$$v_R(x, y) = -2(x - y) + (1 - x - y)$$

By inspection, player 2's minimax payoff is attained by $(1/2, 1/2, 0)$. $\underline{v_2} = 0$.

Theorem 1.2 (Nash Folk Theorem)**.** *For every feasible payoff vector v with $v_i > \underline{v_i}$ for all players i, there exists a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, there is a* ***Nash equilibrium*** *of $G^\infty(\delta)$ with payoff v.*

Proof. (sketch) Consider the following strategy: Play a that achieves v initially, then continue to play a as long as no one deviates (or if there is more than one deviation). If Player j deviates, play m^j (action profile that minimax j) forever. □

Remark. The intuition of this theorem is simply that when the players are patient, any finite one-period gain from deviation is outweighted by being minimaxed in the future (note the strict inequality requirement). The strategies constructed in the proof are "unrelenting." However, such punishment may be very costly for the punishers to carry out. Specifically, they can be not subgame perfect.

Theorem 1.3 ("Nash threats" Folk Theorem)**.** *Let α^* be a Nash equilibrium of the stage game with payoffs e. Then for every feasible payoff vector v with $v_i > e_i$ for all players i, there exists a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, there is a* ***subgame perfect equilibrium*** *of $G^\infty(\delta)$ with payoff v.*

Proof. Change the punishment in Theorem 1.2 from the minimax profile to α^*. □

Theorem 1.4 (Fudenberg and Maskin (1986) Folk Theorem)**.** *Assume that the dimension of the set V of feasible payoffs equals the number of players. Then, for any v with $v_i > \underline{v_i}$ for all players i, there exists a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, there is a* ***subgame perfect equilibrium*** *of $G^\infty(\delta)$ with payoff v.*

Proof. See lecture slides. Intuition:
Phase I: cooperation phase. Play a that achieves v. If a single player j deviates from a, then

play moves to phase II_j.

Phase II_j: minimaxing player j. Play m^j for N periods then move to phase III_j. If player i deviates, move to II_i.

Phase III_j: reward the punishers. Play $a(j)$ forever ($a(j)$ gives slightly more to players other than j). If player i deviates, move to II_i. □

Remark. The various folk theorems show that standard equilibrium concepts do very little to pin down play by patient players.