ECON 6110, Microeconomic Theory III	**Section 6**
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1 Bargaining and the existence

Definition 1.1. Rubinstein's bargaining model consists of:

(i) Two players: $N = \{1, 2\}$

(ii) They bargain over the division of a pie of size one $(\alpha, 1 - \alpha)$, $\alpha \in [0, 1]$

(iii) Individual discount factors, $\delta_i \in (0, 1)$

(iv) In even periods $t = 0, 2, \ldots$

  (a) Player 1 offers $(\alpha_1, 1 - \alpha_1)$

  (b) Player 2 accepts or rejects

  (c) If player 2 accepts, game ends

  (d) Payoffs: $\delta_1^t \alpha_1$ and $\delta_2^t (1 - \alpha_1)$

  (e) If player 2 rejects, game proceeds to the next period

(v) In odd periods $t = 1, 3, \ldots$

  (a) Player 2 offers $(\alpha_2, 1 - \alpha_2)$

  (b) Player 1 accepts or rejects

  (c) If player 1 accepts, game ends

  (d) Payoffs: $\delta_1^t (1 - \alpha_2)$ and $\delta_2^t \alpha_2$

  (e) If player 1 rejects, game proceeds to the next period

Definition 1.2. A strategy of player i is **stationary** if for any history after which it is player i's turn to propose an agreement she proposes the same agreement, and for any history after which it is her turn to respond to a proposal she uses the same criterion to choose her response.

Question 1: Assume stationary strategies. Find a subgame perfect equilibrium.

Remark (Properties of subgame perfect equilibrium).

(i) Immediate agreement: In this subgame perfect equilibria, agreement is reached immediately. This is Pareto efficient.

(ii) First mover advantage:

$$\delta_1 = \delta_2 = \delta \qquad \Longrightarrow \qquad \alpha_1 = \frac{1-\delta}{1-\delta^2} > \frac{1}{2}$$

(iii) Impatience matters: the more impatient a player the worse off she is in equilibrium

$$\frac{\partial}{\partial \delta_1}\left(\frac{1-\delta_2}{1-\delta_1\delta_2}\right) > 0 \quad \text{and} \quad \frac{\partial}{\partial \delta_2}\left(\frac{1-\delta_2}{1-\delta_1\delta_2}\right) < 0$$

Proposition: Consider a two player, infinite-horizon bargaining game and a stationary strategy profile wherein Player i offers $(\alpha_i, 1-\alpha_i)$ and accepts offers by the other player according the payoff threshold $1-\alpha_{-i}$. For any partial history in which i is the acceptor, let z_i denote her expected payoff were play to continue into the next round. Additionally, for any partial history in which i is the offerer, let w_i denote her expected payoff were play to continue into the next round. Then, the stated strategy profile is a SPNE if and only if $z_i = 1-\alpha_{-i}$ and $\alpha_i \geq w_i$.

Proof:($\Longrightarrow$) Suppose that $z_i > 1-\alpha_{-i}$. Then, in any partial history wherein Player $-i$ offers Player i an amount $y \in (z_i, 1-\alpha_{-i})$, i would have a profitable one shot deviation to reject the offer. Conversely, suppose that $z_i < 1-\alpha_{-i}$. Then, in any partial history in which $-i$ offers $y \in (z_i, 1-\alpha_{-i})$, i would have a profitable one shot deviation to accept the offer. Thus, $z_i = 1-\alpha_{-i}$. Finally, suppose that $w_i > \alpha_i$. Then, in any partial history in which i is the offerer, she would have a profitable one shot deviation to propose $(y, 1-y)$ with $y > \alpha_i$. Thus, $\alpha_i \geq w_i$.

($\Longleftarrow$) First, consider a partial history in which i the offerer. Suppose that she has a profitable one shot deviation to offer $(y, 1-y)$ with $y < \alpha_i$. Then, $y > \alpha_i$, contradiction. Conversely, suppose that she has a profitable one shot deviation to offer $(y, 1-y)$ with $y > \alpha_i$. Then, since $-i$ will reject the offer, it must the case that $w_i > \alpha_i$, a contradiction. Now, consider a partial history wherein player i has a profitable one shot deviation to accept some offer $y < 1-\alpha_{-i}$. Then, we must have that $y > z_i$, which implies that $1-\alpha_{-i} > z_i$, a contradiction. Conversely, consider a partial history wherein player i has a profitable one shot deviation to reject some offer $y \geq 1 - \alpha_{-i}$. Then, we must have that $z_i > y$, which implies that $z_i > 1 - \alpha_{-i}$, a contradiction. Since we have found a contradiction for all conceivable one shot deviations, we conclude that the stated strategy profile is a SPNE.

2 Exercises

Consider the alternating-offer bargaining game that we have discussed in class, with the following modification: At the beginning of every period, a coin is toss. If it comes up Heads, player 1 is the initiator who makes the offer. If it comes up Tails, player 2 is the initiator who makes the offer. For this exercise, you may assume that the players have the same discount factor $\delta \in (0, 1)$.

(a) Assume first that the two players agree that the probability of Heads is $p \in (0, 1)$. Find a subgame perfect equilibrium.

(b) Assume now that the two players disagree on the probability of Heads: player 1 believes that the probability of Heads is $p_1 \in (0, 1)$, while player 2 believes that the probability of Tails is $p_2 \in (0, 1)$. The disagreement is commonly known: they agree to disagree. Find a subgame perfect equilibrium.

3 Bargaining and the uniqueness

Question 2: Show that the equilibrium above is the unique SPE.