ECON 6110, Microeconomic Theory III	**Section 4**
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1 Supermodular game

Definition 1.1. $u_i(s_i, s_{-i})$ has (strict) increasing differences in (s_i, s_{-i}) if for all $(s_i, \tilde{s}_i)$ and $(s_{-i}, \tilde{s}_{-i})$ such that $s_i \geq \tilde{s}_i$ and $s_{-i} \geq \tilde{s}_{-i}$, we have:

$$u_i(s_i, s_{-i}) - u_i(\tilde{s}_i, s_{-i}) \geq u_i(s_i, \tilde{s}_{-i}) - u_i(\tilde{s}_i, \tilde{s}_{-i})$$

Definition 1.2. $u_i(s_i, s_{-i})$ is supermodular in s_i if for each s_{-i}:

$$u_i(s_i, s_{-i}) + u_i(\tilde{s}_i, s_{-i}) \leq u_i(s_i \wedge \tilde{s}_i, s_{-i}) + u_i(s_i \vee \tilde{s}_i, s_{-i})$$

Remark. Note that if S_i is linearly ordered (as $\mathbb{R}$), then u_i is trivially supermodular in s_i as the above inequality is vacuously satisfied as equality.

Definition 1.3. A (resp., strictly) supermodular game is a game in which for each i:

- S_i is a sublattice of R^{m_i}
- u_i has (resp., strictly) increasing differences in (s_i, s_{-i})
- u_i is (resp., strictly) supermodular in s_i

Remark. If every players' strategy is single-dimensional, the definition of supermodular game boils down to just increasing differences.

Theorem 1.1. *Let (S, u) be a supermodular game. Then:*

- *the set of strategies surviving iterated strict dominance has greatest and least elements* $\overline{a}, \underline{a}$.
- *and* $\overline{a}, \underline{a}$ *are both Nash equilibria.*

2 Exercise

ECON 6110: 2021 Prelim #1 Question #2

Two students are deciding how long to spend studying for 6110 on the night before the exam. Let e_i be the fraction of the available time student i devotes to studying with $0 \leq e_i \leq 1$. Assume that the students' payoffs are

$$v_1(e_1, e_2) = \log(1 + 3e_1 - e_2) - e_1,$$
$$v_2(e_1, e_2) = \log(1 + 3e_2 - e_1) - e_2.$$

Note: Please ignore the two action profiles that render one of the value functions undefined :)

(a) Show that the game is supermodular.

(b) Find the set of rationalizable actions.

(c) Find the Nash equilibria.