## 1 Best response, strict domination, and rationalizability

**Definition 1.1.** A belief of player *i* about the other players actions is a probability measure over the set  $A_{-i}$ , which we denote as  $\Delta(A_{-i})$ .

*Remark.* Note the difference with  $\prod_{j \neq i} \Delta(A_j)!$ 

**Definition 1.2.** An action  $a_i$  of player i in a strategic game is a **never-best response** if for all beliefs  $\mu_i$  there exists  $\alpha_i \in \Delta(A_i)$  such that

$$u_i(\alpha_i, \mu_i) \ge u_i(a_i, \mu_i)$$

This means that for every belief of player i there is *some* action that is better for player i than  $a_i$ .

**Definition 1.3.** The action  $a_i$  of player *i* in the strategic game is **strictly dominated** if there is *one* mixed action  $\alpha_i$  of player *i* such that for all  $a_{-i} \in A_{-i}$ ,

$$U_i(\alpha_i, a_{-i}) > U_i(a_i, a_{-i})$$

Lemma 1.1 (Osborne & Rubinstein Lemma 60.1).

Never-best response  $\iff$  strictly dominated Sometimes a best response  $\iff$  not strictly dominated (One direction is true only if we allow correlated beliefs.)

*Remark.* One direction is not so obvious!

**Definition 1.4.** An action  $a_i \in A_i$  is **rationalizable** if there exists  $(Z_j)_{j \in N}$  and  $Z_j \subseteq A_j \forall j$  such that:

- $a_i \in Z_i;$
- Every action  $a_j \in Z_j$  is a best response (among  $A_j$ ) to a belief  $\mu_j^{a_j}$  of player j that is supported on  $Z_{-j}$ .

*Remark.* This definition is great for checking whether a given group of sets of strategies is rationalizable, but it doesn't tell us how to find them. The following proposition is thus what we work with most of the time.

**Proposition 1.2.** If  $X = \prod_{j \in N} X_j$  survives iterated elimination of strictly dominated actions in a finite strategic game  $\langle N, (A_i), (u_i) \rangle$  then  $X_j$  is the set of player j's rationalizable actions for each  $j \in N$ . (This is true only if we allow correlated beliefs.)

*Remark.* If we require beliefs to be independent, iterated elimination of strictly dominated strategies leaves a **bigger** set than rationalizable strategies. Reason: when you put restrictions on the belief  $\mu_i(a_i)$ , you can rationalize fewer things. But strictly dominated strategies are not about beliefs and thus not affected.

**Corollary 1.3.** Let  $\alpha^*$  be a mixed Nash equilibrium. For every player *i*, all actions in the support of  $\alpha_i^*$  is rationalizable.

*Remark.* By Proposition 1.2, To find the set of rationalizable strategies, we just need to perform iterated elimination of strictly dominated strategies (IESDS). By Lemma 1.1, we just need to delete the strategies that are never-best response and keep the strategies that are sometimes best response.

## 2 Exercise

## Microeconomics Qualification Exam 2017 Regular: Question IV

Consider a first-price auction featuring two bidders competing for a single object. Bidder 1 values the object at \$1 and Bidder 2 values the object at \$2. After the bidders submit their bids simultaneously, the good is allocated to the winner who has to pay her bid, whereas the loser pays nothing. In the case of a tie, the winner is decided by a fair coin. The rules of auction specify that no bid is allowed to exceed \$5.

(a) What strategies are rationalizable?

(b) Consider a modified version of the game where bids are only allowed in increments of cents (\$.01). What strategies are rationalizable now?