ECON 6110, Microeconomic Theory III	**Section 2**
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1 Correlated equilibrium

Definition 1.1. A correlated equilibrium of a strategic game $\langle N, (A_i), (u_i) \rangle$ consists of:

- a finite probability space (Ω, π) (Ω is a set of states and π is a probability measure on Ω)
- for each player $i \in N$ a partition $\mathcal{P}_i$ of Ω (player i's information partition)
- for each player $i \in N$ a strategy $\sigma_i : \Omega \to A_i$ with $\sigma_i(\omega) = \sigma_i(\omega')$ whenever $\omega, \omega' \in P_i$ for some $P_i \in \mathcal{P}_i$ (equivalently, $\sigma_i : \mathcal{P}_i \to A_i$)

such that for every $i \in N$ and every alternative strategy $\tau_i : \Omega \to A$ of player i that satisfies $\tau_i(\omega) = \tau_i(\omega')$ whenever $\omega, \omega' \in P_i$, we have

$$\sum_{\omega \in \Omega} \pi(\omega) u_i(\sigma_i(\omega), \sigma_{-i}(\omega)) \geq \sum_{\omega \in \Omega} \pi(\omega) u_i(\tau_i(\omega), \sigma_{-i}(\omega)) \tag{1}$$

or

$$\sum_{\omega \in \Omega} \Pr(\omega \mid P_i) u_i(\sigma_i(\omega), \sigma_{-i}(\omega)) \geq \sum_{\omega \in \Omega} \Pr(\omega \mid P_i) u_i(\tau_i(\omega), \sigma_{-i}(\omega)) \quad \forall P_i \in \mathcal{P}_i \tag{2}$$

Proof. (2) $\implies$ (1): Law of total expectations.

(1) $\implies$ (2): Proof by contradiction. $\tau_i(\omega) = \sigma_i(\omega), \forall \omega \notin P_i$ and $\tau_i(\omega'), \forall \omega' \in P_i$. $\square$

Proposition 1.1. *WLOG can assume $\Omega = A = \Pi_i A_i$, $\mathcal{P}_i$ consists of sets of the type $\{a \in A : a_i = b_i\}$ for some action $b_i \in A_i$, and $\sigma_i(a) = a_i$.*

Remark. The initial π would give the final distribution of outcomes if the construction is successful.

Remark. The probability space and the partitions are endogenous, part of the equilibrium definition.

Remark. Correlated equilibria can be found with linear programming. When all players in the game are *no-SWAP regret* algorithms, the empirical distribution of the play converges to a correlated equilibrium.

2 Exercise

In the following game, compute all the Nash equilibria, and find a correlated equilibrium that is not in the convex hull of the Nash equilibria.

	L	R
U	$8, 8$	$4, 9$
D	$9, 4$	$1, 1$